Institute of Actuaries of India

Solutions for ACET 2017

Mathematics

1. C. $\cos 135° = \cos(90° + 45°) = \cos 90° \cos 45° - \sin 90° \sin 45° = -\frac{1}{\sqrt{2}}$.

2. B. The equation $x^2 - 3x - 18 = 0$ gives $(x-6)(x+3) = 0$.
   Hence the inequality $x^2 - 3x - 18 > 0$ gives
   either $(x-6) > 0$ and $(x+3) > 0$ OR $(x-6) < 0$ and $(x+3) < 0$.
   This gives $x > 6$ and $x > -3$ OR $x < 6$ and $x < -3$.
   This implies $x > 6$ OR $x < -3$, that is, $x \in (-\infty, -3) \cup (6, \infty)$.

3. A. $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ implies $|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$ which again implies
   $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$.
   Thus we have $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$.
   That is, $\vec{a} \cdot \vec{b} = 0$ and $\vec{a}$ is perpendicular to $\vec{b}$.

4. B. If $ax^2 + bx + c = 0, a \neq 0$, has equal roots, then $b^2 - 4ac = 0$.
   Hence, $c = \frac{b^2}{4a}$.

5. D. $\frac{1}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$.
   This implies $1 = A(x-2)^2 + B(x-1)(x-2) + C(x-1)$.
   Putting $x = 2$, we get $C = 1$ and on putting $x = 1$, we get $A = 1$.
   Equating the coefficients of x^2 we have $A + B = 0$ giving $B = -1$.
   Thus , $\frac{1}{(x-1)(x-2)^2} = \frac{1}{x-1} - \frac{1}{x-2} + \frac{1}{(x-2)^2}$.

6. D. $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{n+6} = \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n \times \left(1 + \frac{1}{n}\right)^6 = e$.

7. B. $y = \frac{\log x^2}{e^x}$, so $\frac{dy}{dx} = \frac{e^x \frac{d(\log x^2)}{dx} - \log x^2 \frac{de^x}{dx}}{(e^x)^2} = \frac{e^x \frac{2x}{x^2} - \log x^2 e^x}{(e^x)^2} = \frac{\frac{2x}{x^2} - \log x^2}{e^x} = 2e^{-x}\left(\frac{1}{x} - \log x\right)$.
   Alternatively, $y = e^{-x}(\log x^2) = 2e^{-x}(\log x)$, ie, $\frac{dy}{dx} = -2e^{-x}(\log x) + 2e^{-x}\left(\frac{1}{x}\right)$.

8. C. $y = \tan^{-1}(\log x)$, put $u = \log x \to y = \tan^{-1} u$. By Chain rule,

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{1+u^2} \cdot \frac{1}{x} = \frac{1}{x(1 + (\log x^2))}.$$

9. A. $f(x) = 1 - x - x^2$, then $f'(x) = -1 - 2x$, which is negative if $x < -\frac{1}{2}$. Hence, $f(x)$is increasing in $\left(-\infty, -\frac{1}{2}\right)$.

10. A. $\int x\cos x\,dx = \int x d(\sin x) = x\sin x - \int \sin x\,dx = x\sin x + \cos x + c$.

11. A. $\int_0^\infty x^7 e^{-\frac{x}{2}}\,dx = \int_0^\infty (2y)^7 e^{-y} 2dy$ (where $\frac{x}{2} = y$) $= 2^8 \int_0^\infty y^{8-1} e^{-y} dy = 2^8 7!$

12. C. If $f(x) = \sin x\ \cos^4 x$, then $f(-x) = \sin(-x)\cos^4(-x) = -\sin x\ \cos^4 x = -f(x)$. Since $f(x)$ is an odd function, $\int_{-1}^{1} \sin x\ \cos^4 x\,dx = 0$.

13. D. If $P = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$, then $P^2 = I$ implies $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = I$, that is, $\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. This implies $1 - \alpha^2 - \beta\gamma = 0$.

14. B. $P = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $P^T = [1\ \ 2\ \ 3]$. Therefore, $PP^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$. The determinant is 0. All 2×2 submatrices also have determinant 0. Therefore, the rank is 1.

15. D. If $P = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$, then $|P| = 2(6-2) - 2(2-1) + 1(2-3) = 5$.

$Adj\ P = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix}$. Hence, $P^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -2 & -1 \\ -1 & 3 & -1 \\ -1 & -2 & 4 \end{bmatrix}$.

16. A. Given that $\log_e 1.5 = 0.405$. Also, $\log_e 1 = 0$.
We have two points: $(x_0, y_0) = (1, 0)$ and $(x_1, y_1) = (1.5, 0.405)$
Linear interpolate of $\log_e 1.3$,

$$y = \frac{y_0(x_1 - x) + y_1(x - x_0)}{x_1 - x_0} = \frac{0 + 0.405(1.3 - 1)}{0.5} = \frac{0.3 \times 0.405}{0.5} = 0.6 \times 0.405$$
$$= 0.243.$$

17. D. $0.99^4 = (1 - 0.01)^4 = C_0^4\, 1^4 - C_1^4\, 1^3\,.01^1 + C_2^4\, 1^2\,.01^2 = 1 - 4 \times 0.01 + (6 \times 0.0001)$ $= 1 - 0.04 + 0.0006 = 0.9606$.

Statistics

18. C. $C_3^8 = 56$.

19. D. $P(R) = P(R|B_1)P(B_1) + P(R|B_2)P(B_2) = \frac{5}{8} \times \frac{2}{3} + \frac{4}{10} \times \frac{1}{3} = \frac{33}{60}$.

20. C. $P(E \cup F) = \frac{8}{15} - \frac{1}{15} = \frac{7}{15}$, $P(E \cup F) = \frac{8}{15}$, $P(E \cap F^c) = \frac{1}{5} \times \frac{2}{3} = \frac{2}{15}$, $P[(E \cup F)^c] = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

21. **B.** $123 + 7p = 6(22 + p)$. This implies $p = 9$.

Value	3	4	5	6	7	8	9
Frequency	1	4	7	6	9	3	1
Cum freq	1	5	12	18	27	30	31

Total frequency = 31. Mode = 7, Median = 16^{th} observation = 6.

22. B.

23. D. $E(X^{11}) = 1. P(X = 1) = 0.7$.

24. C. $np = 4$, $npq = 2.4$. This implies $q = 0.6, p = 0.4$, $n = 10$. $P(X \geq 1) = 1 - 0.6^{10}$.

25. D. $X \sim$ Poisson(2).

$$E[X(X-1)(X-2)(X-3) = \sum_{x=0}^{\infty} x(x-1)(x-2)(x-3)\frac{e^{-2}2^x}{x!} = \sum_{x=4}^{\infty} \frac{e^{-2}2^x}{(x-4)!}$$

$$= 2^4 = 16.$$

26. **B.** $f(x) = \frac{1}{12}$, $E(X) = \frac{12}{2} = 6$, $Var(X) = \frac{12^2}{12} = 12$. CV = $\sqrt{Var(X)}/E(X) = \sqrt{3}/3$.

27. B. $a = E(X) = \frac{1}{3}$.

28. A. $P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) = P\left(-2 \leq \frac{X-\mu}{\sigma} \leq 2\right) = \Phi(2) - \Phi(-2) = 2\Phi(2) - 1 = 2\big(0.5 + P(2)\big) - 1 = 2\,P(2)$.

29. B. $Cov(X, X^2) = E(X^3) - E(X)E(X^2) = 0 - 0 \times 1 = 0$.

30. C. $\bar{x} = 10, \quad \bar{y} = 7.$

x	1	8	10	10	14	17
y	1	4	6	12	12	7

The regression of y on z is

$$y = \bar{y} + b_{yx}(x - \bar{x}).$$

Intercept $= \bar{y} - b_{yx}\,\bar{x} = 7 - 0.533 \times 10 = 7 - 5.33 = 1.67$. Thus,

$$y = 7 + 0.533(x - 10) = 7 + (0.533/3)(3x - 30).$$

The regression of y on z is

$$y = 7 + (0.533/3)(z - 30),$$

which has slope $0.533/3$ and intercept $7 - (0.533/3) \times 30 = 1.67$.

Data Interpretation

31. A. Number of students $= 3 + 6 + 8 + 20 = 37$.

32. C. Number of students $= 35 + 12 + 8 = 55$.

33. B. Number of students $= -3 + 12 + 8 + 5 + 3 = 25$.

34. D. Between $3 + 6 + 8 = 17$ and $3 + 6 + 8 + 20 = 37$.

35. B. There are $8 + 5 + 3 = 16$ students with score above 69 and $16 + 12 = 28$ students with score above 59. Since 25 lies in between 16 and 28, the 75$^{\text{th}}$ smallest score is in between 60 and 69.

36. C. Number of candidates passed *only* in Mathematics $= 80 - 18 - 30 - 12 = 20$.
Number of candidates passed in both English and Mathematics is 30.
Number of candidates passed in Mathematics $= 20 + 30 = 50$.

37. C. Sales increase in 1992-1993 is $\frac{15}{25} \times 100 = 60\%$. It can be visually checked that percentage increases in the other years are smaller.

38. A. $\frac{150}{20} \times 100 = 750\%$.

English

39. A

40. A

41. C

42. D

43. B

44. …

45. D

46. A

47. C

48. D

49. B

50. D

51. B

52. A

53. A

54. B

55. D

56. B

57. C

58. D

59. B

60. D

61. C

62. D.

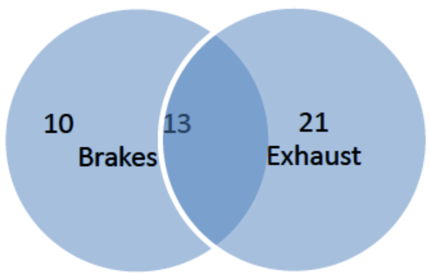
Logical Reasoning

63. D. The doctors who are fools may not be rich; so conclusion I does not hold. On the other hand, the fools who are rich may not be doctors; so conclusion II does not hold.

64. C. Mohan is the only person returning to a social system that he has been away from for an extended period of time.

65. B.

66. A. Number of cars that needed new brakes or new exhaust system is $50 - 6 = 44$. Number of that needed new exhaust system is 34. Therefore, number of cars that needed new brakes but no new exhaust system is $44 - 34 = 10$.



67. D. The father of the girl's uncle → the grandfather of the girl. Daughter of the grandfather → either mother or aunt. Therefore, the boy is the girl's brother or cousin.

68. B. After every 400 years, the same day occurs. Thus, if 27th February 2003 is Thursday, before 400 years i.e., on 27th February 1603 must have been a Thursday too.

Alternative argument: 365 is $7 \times 52 + 1$. Therefore, in a non-leap year, 27th February falls on the preceding day of the week as compared to the following year. In other words, a day of the week is lost in a non-leap year.

An extra day is lost in a leap-year.

Every 4th year is generally a leap-year. There are 100 multiples of four between 1603 and 2003.

Every 100th year is an exception to the leap-year rule. There are 4 multiples of 100 between 1603 and 2003.

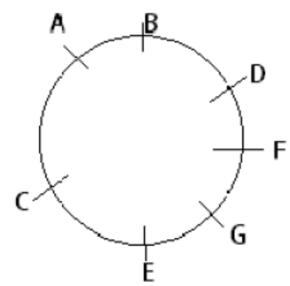
Every 400th year is an exception of these exceptions. There is only one multiple of 400 between 1603 and 2003.

Therefore, the number of days lost is $400 + 100 - 4 + 1 = 497$, which is a multiple of 7. Hence, no day of the week is lost. 27th February 1603 must have been a Thursday.

69. B. The number of seconds lost in the first week is $0.01 \times 7 \times 24 \times 60 \times 60 = 6048$. The number of seconds gained in the first week is $0.02 \times 7 \times 24 \times 60 \times 60 = 2 \times 6048$. Net gain is 6048 seconds, i.e., 1 hour 40 minutes 48 seconds.

70. A. The fourth clue gives the sequence E G F or F G E. The third clue suggests E G F D or F G E D, but the first clue rules out F G E D. So we definitely have E G F D.

A, B and C must be to the right of D in some order. The second clue gives the sequence B A C or C A B, but the first clue rules out the latter sequence to occur to the right of D.



Thus, we have E G F D B A C.

B is between A and D, and none of the other statements is correct.

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